**2023 May Day MCM**

**Problem A. The problem of fixed-point airdrop of unmanned aerial vehicle**

With the development of science and technology, UAVs (unmanned aerial vehicles) are widely used in many fields. The delivery accuracy of a fixed-point airdrop of a UVA depends not only on the operating technique, but also on the state and environment in which UAV performs the task. For example, the height and speed of the UAV near the drop point, wind speed at the location of the UVA, the geographical environment around the drop point of the UVA, and other factors. In this subject, considering only jet UAVs, please consult relevant information and research the following questions:

**Question 1**：Assume that the UAV flies parallel to the horizontal plane and drops supplies (spherical, radius 20cm, weighs 50 kg) to the specified position on the ground.

1. Please establish a mathematical model, and give the relationship between drop distance (the straight-line distance between the UAVs and the designated drop point for supplies), the height of the UAV, the speed of the UAV, air resistance, etc.
2. It is assumed that the height of the UAV is 300m, the speed is 300km/h, the wind speed is 5m/s, and the direction of the wind is parallel with the horizontal plane. Please establish a mathematical model, and give the drop distance of the UAV when the flight direction is the same as the wind direction (angle is 0 degrees), opposite (angle is 180 degrees), and vertical (angle is 90 degrees).

**Question 2**：UAVs can drop supplies to a specified position on the ground and launch explosives to unblock waterways. The general process is as follows: First, the UVA approaches the area of the obstacle horizontally and dives to launch an explosive. Then, the UAV lifts and fly away when the launch is complete. Now a river is blocked up by an ice block and needs to launch an explosive to unblock waterways (suppose the explosive is spherical, radius 8cm, and weighs 5kg). Assume that the horizontal distance from the initial point of the UVA to the target is 10000m. Due to the environmental impact, the UAV has to dive to launch in the same direction as the UAV flies.

1. Please establish a mathematical model, and give the relationship between the launch distance (the straight-line distance between the launch point of the UAV and the target) and the height of the UAV, the speed of the UAV, the dive angle, the launch speed, etc.
2. It is assumed that the wind speed is 6m/s, the height of the UAV is 800m when approaching the target, the flight speed of the UAV is 300km/h, and the launch speed of the explosive is 600km/h (relative to the speed of UAV). The distance between the UAV and the target is required to be between 1000m-3000m when the UAV launches the explosive. Please give the launch strategy of the UAV for hitting the target.

**Question 3**：The accuracy of the UAV launching explosives to hit the target has a great relationship with the stability of the flight of the UAV. Under the same conditions, the more stable the UAV is when launching explosives, the more accurately hitting the target. After the dive begins, the operator needs to constantly adjust the attitude of the UAV to correct the effect of wind direction and speed on the UAV.

1. Under certain flight and launch speeds, considering various factors, please establish a mathematical model to quantify the flight stability of the UAV and give the relationship between the stability and the hitting accuracy, and analyze and verify the stability of the UAV by numerical simulation methods.
2. It is assumed that the wind speed is 6m/s, the flight speed range of the UAV is 300 km/h to 400 km/h, and the launch speed of the explosive is 500km/h (relative to the speed of the UAV). The UAV begins to dive at an altitude of 800m with an initial dive angle of 45 degrees; the height of the UAV is not less than 300m when the UAV launches explosives. Please give the optimal attitude adjustment strategy to keep the stability of the UAV as much as possible.